STRAIGHT LINE

Things of science

Unit No. 235

received by a Group of Friends of Science: sponsored and distributed without profit by Science Service, the Institution for the Popularization of Science, 1719 N Street N. W Washington 6, D. C. Watson Davis, Director

This unit of THINGS of science consists of ten specimens of bars plus the necessary fasteners needed to make four linkages, and this explanatory leaflet. With these linkages you can make your own original straight line and other curves.

There is probably no geometrical construction that is so readily accepted without questioning "why?" as that of drawing a straight line.

That it is possible to draw a straight line seems at first to be self evident. However, you know that you can scarcely draw a straight line freehand. Therefore, you will no doubt reach for a ruler or other straightedge. You lay the ruler on the paper and trace along it. If the ruler is straight, your line will be straight. But it is not your line. You copied it from the edge of the ruler.

Ordinary plane geometry is based on the assumption that we can get straight lines and circles at will. As we shall see, we can draw a circle at will with any point as center and any given distance as radius. But the straight line is not so simple.

The problem, then, is to devise a mechanism that will draw a straight line, but will not necessitate the use of a straight line in its own construction.

Although Euclid postulated the straight line in 300 B. C., it was not until over 2,000 years later that a mechanical device was invented for constructing an original straight line in a plane.

Euclidian geometry is based upon the two fundamental theories: first, that it is possible to draw a straight line through any two points; second, that it is possible to draw a circle with any point as center and any line segment as radius.

DRAWING A CIRCLE IS EASY

The problem of drawing your own original circle is quite simple. All you need is any object which will not stretch or compress. If you fix one point of it and let the object rotate in a plane about the fixed point, any other point on the object will trace a perfect circle. This, of course, is the principle of the compass.

Thus, if you fasten a piece of cardboard to a drawing board with a thumb tack, insert a pencil point in a hole anywhere in the cardboard, and move the pencil and the cardboard with the thumb tack acting as a pivot, you will get a perfect circle. Or if you tie the pencil to the tack with a string and move the pencil, keeping it perpendicular to the board and the string taut, you will get a circle. If we drew circles in the way we draw straight lines, by copying them, we would have a series of cut-out patterns of circles around which we would trace.

Although mathematicians had been using the straight line for many centuries, it was the pressure of practical mechanics that that brought about the first method of tracing an original straight line in a plane -- by means of a linkage.

This unit contains the following specimens necessary to construct four linkages that provide means of making an original straight line and two other curves.

STRAIGHT LINE CLASSIFIED AS CURVE

To classify the straight line as a curve may seem strange to some persons, but it is correct. The word "curve" refers to a class of figures of which the straight line is one member.

Identify the specimens as you break them from the die-cut cardboard. The measurement is the distance between the centers of the holes in the bars.

- 3 - 4-inch bars
- 4 - 2 1/2-inch bars
- 2 - 1 9/16-inch bars
- 1 - 1 1/4-inch bar

Also included in this kit are one 3 x 5-inch hole-punched cardboard backing, 10 paper fasteners, 10 eyelets, and 12 washers.

ASSEMBLING THE LINKAGES

Four linkages can be put together with the bars, eyelets and paper fasteners contained in this kit. You can easily tell by studying the drawing of each linkage how the bars should be assembled at each joint, which bar should be on top and which underneath. Push a round eyelet down through the holes to make each joint.

On the underside of the joint put one or two cardboard washers. Use whatever number is needed to make a good pivot. Insert a paper fastener through the eyelet and spread out the prongs of the fastener on the underside.

The eyelet then acts as a pivot, and the paper fastener and the washer simply keep the bars from sliding off the eyelet. This arrangement should give you smoothly working pivots with practically no play. By straightening out the paper fastener, you can remove it and thus disassemble the bars to make a different model.

When a joint is to be fastened to the 3 x 5-inch cardboard base, you should push the eyelet through the base. You will note that the 3 x 5 base has five holes. If you will number these 1, 2, 3, 4 and 5 on your card, it will aid you in assembling the models. The models should always be assembled exactly as indicated in each drawing. Your card should be numbered so that if the holes are numbered from left to right, those with numbers 2 and 3 are the two quite close together.

WATT'S LINKAGE

James Watt, one of the inventors of the steam engine, made the first linkage. Watt needed a mechanical device that would guide the piston of a steam engine in an exact straight line. In an attempt to achieve this result, he devised the

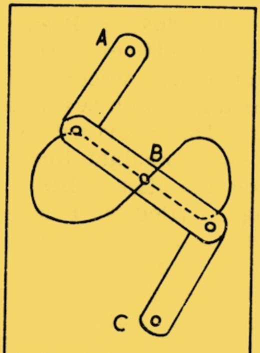

Figure I

first known plane linkage. To assemble Watt's linkage, take from the kit the two bars which measure 1 9/16-inch between the centers of the holes and the 2 1/2-inch bar which has a hole punched in the center. Assemble them as shown in Figure I.

The linkage must be pivoted to holes 1 and 5 in the 3 x 5 base.

To preserve your 3 x 5 cardboard backing, cover the center portion with a piece of paper before tracing your first curve by means of a linkage.

If now you hold the point of a pencil in the hole indicated by B, the midpoint of the middle bar, and deform the linkage, it will trace for you a figure-eight curve. No part of this curve is truly straight. However, the parts near the crossing point or node, are nearly straight. You can easily see this after you make the linkage and trace your figure eight.

Using this type of linkage, Watt was able to attach the piston of his engine to the point B of the linkage. If the stroke of the engine was not too long, the point B remained on the straighter part of the curve and guided the piston in a nearly straight line. Watt developed this linkage in 1782.

THE PEAUCELLIER CELL

In 1864, A. Peaucellier, an officer in the French army, discovered the linkage which bears his name. This provided the first exact straight-line motion in a plane. His linkage, called the Peaucellier Cell, consists of a rhombus (a plane figure with four equal sides) with two other bars attached.

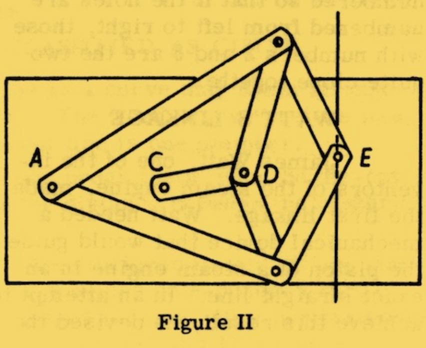

Figure II

Use the four 2 1/2-inch bars for the rhombus, with two of the 4-inch bars, as shown in Figure II.

The point A is pivoted to hole 1. The bar CD is a 1 1/4-inch bar and is pivoted to hole 3. Its purpose is to make the point D follow a circle. This circle would pass through A if the linkage permitted D to move that far.

The remarkable property of the Peaucellier Cell is that the product of the distance AD and the distance AE is always a constant. You can verify this experimentally by measuring AD and AE accurately with the linkage in a variety of positions. If you multiply together the two measurements for any given position, the result will always be approximately 9.75.

Because of this property, when the point D is made to follow any circle passing through A, the point E must follow an exact straight line. Since the cell carries out the transformation known in modern geometry as inversion, this linkage is a mechanical inversor and can be used to illustrate a number of the theorems on inversion.

For instance, pivot the point C in hole 2. Then D follows a circle which cannot pass through A, and E now traces an arc of another circle.

ANOTHER STRAIGHT-LINE LINKAGE

You will need three 4-inch, three 2 1/2-inch, and one 1 9/16-inch bars to assemble this linkage. Mount the bars EF and FG under the others so that the point F can move under the bar BC. Points A and D are pivoted through holes 1 and 5.

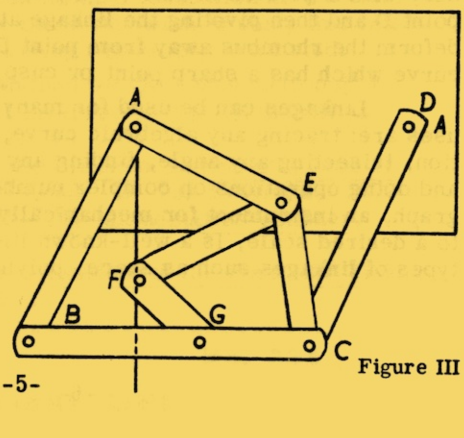

Figure III

Now when the linkage is deformed, F traces a portion of an exact straight line. The principle here is quite different from that used to produce a straight line in the previous model.

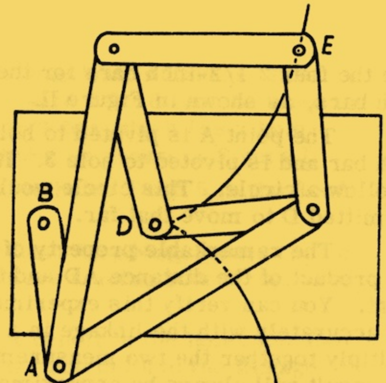

Figure IV

THE CISSOID CURVE

All the linkages so far have been connected with the straight line. However, there are many other curves of interest in mathematics. One of these, called the Cissoid of Diocles, can be traced by the use of a Peaucellier Cell.

The general arrangement of the cell is like that used in the second model, except that it will be best to place the two bars of the rhombus which form the outside bars of the figure on top of the other bars. Then pivot point D in hole 4. AB is a 1 9/16-inch bar, with B pivoted in hole 5. By deforming the linkage towards point D and then pivoting the linkage at point D and continuing to deform the rhombus away from point D, the linkage will trace a curve which has a sharp point or cusp at D.

Linkages can be used for many purposes. Some of the uses are: tracing any algebraic curve, solving any algebraic equation, trisecting any angle, finding any root or power of a number, and doing operations on complex numbers. The common pantograph, an instrument for mechanically copying maps according to a desired scale, is a well-known linkage. Also there are other types of linkages such as space, polyhedral and spherical linkages.

STRAIGHT LINE NOT NEEDED

The argument is sometimes advanced that linkages do not really solve the straight line problem because we must first be able to draw a straight line in order to construct the linkage. This argument, of course, is not valid. No straight line is needed to construct any linkage. The bars of a linkage are usually made straight merely for the sake of appearance. But they could just as well be made of crooked or curved pieces. It is only necessary that the distance between pivots be laid off in certain ratios, and these distances can all be constructed with the compass alone.

You can test this statement yourself by cutting from stiff cardboard pieces that have the same distance between pivoting points, but do not have straight edges. You can make the edges either circular, wavy, or whatever particular form suits your fancy.

FURTHER STUDY

The study of linkages is an interesting branch of mathematics. No attempt has been made in this kit to prove mathematically the statements made. However, no mathematician will decide that a line is straight merely because it looks straight. The theories in linkages have as sound a foundation as the other theories of mathematics. For those who wish to study this interesting subject further, the following references are listed.

Ahrendt, M. H., "A General Method for the Construction of a Mechanical Inversor" - The Mathematics Teacher, 37, 76.

De Roos, J. D. C., Linkages, the Different Forms and Uses of Articulated Links D. Van Nostrand. 1879

Hilsenrath, J., "Linkages" - The Mathematics Teacher, 30, 277.

Hrones, J. A. and G. L. Nelson, Analysis of the Four Bar Linkage, Wiley. 1951.

Kempe, A. B., How to Draw a Straight Line Macmillan & Co. 1877.

Meserve, B. E., "Linkages as Visual Aids" - The Mathematics Teacher, 39, 372.

Yates, R. C., "The Story of the Parallelogram" - The Mathematics Teacher, 33, 301.

Yates, R. C., "Linkages" - Eighteenth Yearbook of the National Council of Teachers of Mathematics, p. 117. Bureau of Publications, Columbia University, New York. 1945.

Yates, R. C., Geometrical Tools. Educational Publishers, St. Louis. 1949.

Because of the enthusiastic response and the continued demand for the Straight Line Unit, which was originally issued in March, 1950, with the cooperation of the National Council of Teachers of Mathematics, this unit is being reissued.

* * *

Production by Jane Marye

Unit No. 235 Issued May 1960